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Polarization of Čerenkov Radiation in Anisotropic Media

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Using the method of Stokes parameters, we examine the polarization of Čerenkov radiation in anisotropic media. The study reveals that the radiation is totally polarized and that circular polarization is purely a quantum effect. We examine two cases: when the particle initially moves along the optical axis and when the particle initially moves perpendicular to the optical axis.

1. INTRODUCTION

Despite extensive literature on the topic, the phenomenon of Čerenkov radiation remains a subject of interest. The fact that it has found application in technology has accentuated this interest. Although most of these studies are devoted to the case of isotropic media, interest has extended to other types of media. Studies devoted to anisotropic media (Ginsburg, 1940; Kukanov and Orisa, 1971) meet with greater problems because of the complicated nature of the radiation. An aspect of the phenomenon of particular interest is the polarization. Tamm and Frank (1937), who were the first to give a theoretical explanation of the phenomenon, showed that the radiation has a total linear polarization. However, Loskutov (1960), using the methods of quantum electrodynamics, showed that there is a circularly polarized component. For the degree of circular polarization he obtained the expression

$$P_{\text{cir}} = \frac{sn(\omega\hbar/cp)[1 - (\cos\theta)/\beta n]}{\sin^2\theta + (n\,\omega\hbar/2cp)^2(1 - n^{-2})} \tag{1}$$

where $s = \pm 1$ is the polarization of the electron. In the concluding section an explanation will be proferred for this seemingly insufficient picture given

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by Tamm and Frank and even by the experiments of the discoverer Čerenkov himself.

No detailed study of the polarization in anisotropic media is available. Ginsburg (1940), using what he called the Hamiltonian method, studied the phenomenon in uniaxial crystals and remarked that there is a substantial difference in the pattern and polarization of the radiation compared with those of isotropic media. The nature of such differences, particularly in the polarization, was, however, not analyzed.

In this paper, using the powerful tools of the Stokes parameters as a kind of Nicol prism, we carry out a more detailed analysis of the polarization of Čerenkov radiation in anisotropic media. In order to obtain a broader picture we shall use the methods of quantum electrodynamics. In the next section we briefly obtain an expression for the intensity of radiation since this is relevant for our analysis. Although the determination of the intensity has been done elsewhere, we need to write it in a form which will involve the parameters necessary to assist us with our analysis. In Section 3 the Stokes parameters are introduced and applied. Section 4 discusses the results. A clearer picture is obtained by giving definite values to the parameters of the problem.

2. INTENSITY OF RADIATION

If we consider the case of a medium which is both electroactive and magnetoactive, Maxwell's equations in a medium in terms of the vector potential (scalar gauge) can be shown to reduce to the following differential equation:

$$\left(e_{\alpha\beta\gamma}\mu_{\gamma\lambda}^{-1}e_{\lambda\delta\nu}\frac{\partial^2}{\partial x_\beta \partial x_\delta} + \frac{1}{c^2}\epsilon_{\alpha\beta}\frac{\partial^2}{\partial t^2}\right)A_\nu = 0 \tag{2}$$

In order to study quantum processes in a medium Ginsburg (1940) and Sokolov (1940) developed methods based on a phenomenological approach. Both of them used the methods so developed to study radiation processes in media. Alexeev and Nikitin (1965, 1966) generalized their methods to the case of anisotropic media. Following their footsteps, we express the solution of (2) in terms of plane waves in the form

$$\mathbf{A} = L^{-3/2}\sum_{q,j}\left(\frac{4\pi c^2\hbar}{\mathcal{M}}\right)^{1/2}(\mathbf{a}e^{-i\omega t+i\mathbf{q}\cdot\mathbf{r}} + \mathbf{a}^+e^{i\omega t-i\mathbf{q}\cdot\mathbf{r}}) \tag{3}$$

As a result of the quantization of the electromagnetic field, the amplitudes $\mathbf{a}$ and $\mathbf{a}^+$ now acquire the dual properties of classical and quantum numbers. Their classical parts are three-component vectors defining the polarizations of the transverse and longitudinal photons. Here we shall be interested in the polarization of the transverse photons only. On the other hand, in view of their quantum nature, they satisfy the commutation relation

$$\mathbf{a}\mathbf{a}^{+} - \mathbf{a}^{+}\mathbf{a} = 1 \tag{4}$$

It has been shown that equation (2) has two solutions (eigenvectors) corresponding to two types of waves. In a magnetoactive medium the two waves are transverse. Their polarization is easily determined. In an electroactive medium, however, one wave is transverse, the other is not. We shall therefore be interested in this latter case. We therefore put $\mu_1 = \mu_3 = 1$ and

$$\epsilon_{\alpha\beta} = \begin{pmatrix} \epsilon_1 & 0 & 0 \\ 0 & \epsilon_1 & 0 \\ 0 & 0 & \epsilon_3 \end{pmatrix}$$

For a study of the polarization of the transverse photons, we write the amplitude in the form

$$\mathbf{a} = \mathbf{a}_f f + \mathbf{a}_g g$$

where the vectors $\mathbf{a}_f$, $\mathbf{a}_g$ are classical parts, and f and g provide the quantum parts of $\mathbf{a}$.

Further,

$$\mathbf{a}_f = \cos\alpha\, \boldsymbol{\beta}_2 + e^{i\gamma} \sin\alpha\, \boldsymbol{\beta}_3 \tag{5}$$

$$\mathbf{a}_g = \sin\alpha\, \boldsymbol{\beta}_2 - e^{i\gamma} \cos\alpha\, \boldsymbol{\beta}_3 \tag{6}$$

$\boldsymbol{\beta}_2$ and $\boldsymbol{\beta}_3$ are unit vectors defined as

$$\boldsymbol{\beta}_2 = \frac{\boldsymbol{\kappa} \wedge \mathbf{k}^0}{[1 - (\boldsymbol{\kappa}\cdot\mathbf{k}^0)^2]^{1/2}}; \qquad \boldsymbol{\beta}_3 = \boldsymbol{\kappa} \wedge \boldsymbol{\beta}_2$$

where

$$\boldsymbol{\kappa} = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta) \tag{7}$$

is the unit vector in the direction of the wave vector $\mathbf{q}$ and $\mathbf{k}^0 = (0, 0, 1)$ is directed along the optical axis. Note that $\boldsymbol{\beta}_2$, $\boldsymbol{\beta}_3$, $\boldsymbol{\kappa}$ form an orthonormal triad.

Assuming that initially there are no photons in the field, in (4) we may take $aa^{+} = 1$, $a^{+}a = 0$. Following from this we take

$$ff^{+} = gg^{+} = 1, \qquad f^{+}f = g^{+}g = 0$$

In a medium, the wave vector is related to the frequency by

$$\mathbf{q} = n\frac{\omega}{c}\boldsymbol{\kappa} \tag{8}$$

where n is the refractive index, and in this case

$$n_{+1}^2 = \epsilon_1$$

$$n_{-1}^2 = \frac{\epsilon_1 \epsilon_3}{\epsilon_1 \sin^2\theta + \epsilon_3 \cos^2\theta}$$

The corresponding eigenvectors are given by

$$\mathbf{l}_{+1} = (\sin \varphi, -\cos \varphi, 0) \tag{9}$$

$$\mathbf{l}_{-1} = \frac{(\epsilon_3 \cos\theta \cos\varphi, \epsilon_3 \cos\theta \sin\varphi, -\epsilon_1 \sin\theta)}{(\epsilon_1{}^2 \sin^2\theta + \epsilon_3{}^2 \cos^2\theta)^{1/2}} \tag{10}$$

It is easy to verify that

$$\mathbf{l}_{+1} \cdot \boldsymbol{\kappa} = 0; \qquad \mathbf{l}_{-1} \cdot \boldsymbol{\kappa} \neq 0$$

We write the operator of the energy of interaction in the form

$$\mathscr{E} = e(\mathbf{a} \cdot \mathbf{A}) \tag{11}$$

where e is the charge on the particle and $\boldsymbol{\alpha}$ is 4×4 Dirac matrix. Following Sokolov (1958), using perturbation methods, we find the probability of transition

$$\omega_j = \frac{2\pi}{c\hbar^2} \sum_{k'} \sum_{q} R_j^+ R_j \delta_{\mathbf{k},\mathbf{k}'+\mathbf{q}} \delta\left(K' + \frac{\omega}{c} - K\right) \tag{12}$$

where $R_j = L^{-3/2}(4\pi c^2 \hbar / \mathcal{M})^{1/2} \Gamma_j$, with

$$\Gamma_j = \left(1 + ss' \frac{kk'}{KK'} - \frac{k_0^2}{KK'}\right)\left[\left(1 - ss' \frac{\mathbf{k}\cdot\mathbf{k}'}{kk'}\right)\mathbf{a}\mathbf{a}^+ + \frac{i}{kk'}(sk'\mathbf{k} - s'k\mathbf{k}')\mathbf{a} \wedge a^+ + ss' \frac{(\mathbf{a}\cdot\mathbf{k}')(\mathbf{a}^+\cdot\mathbf{k}) + (\mathbf{a}\cdot\mathbf{k})(\mathbf{a}^+\cdot\mathbf{k}')}{kk'}\right]$$

In the above expression $\hbar k = p$, $c\hbar K$ and $\hbar k'$, $c\hbar K'$ are respectively the momentum and energy of the particle. The eigenvalues of the projection of the spin operator along the momentum are given by $\frac{1}{2}\hbar s$, $\frac{1}{2}\hbar s'$ ($s, s' = \pm 1$). Quantities without a prime refer to the initial state. Those with a prime refer to the final state. L is the dimension of an arbitrary elementary cube. We can now write down the intensity of radiation using (5), (6), and (12). We consider two cases.

Case I. Particle initially moves along the optical axis. In this case $k = k(0, 0, 1)$. The expression (12) is independent of φ. Integration with respect to φ therefore leads to a factor 2π only. Integration with respect to θ is effected by using the properties of the δ-function of complex argument.

The summation with respect to k' gives

$$\mathbf{k}' = \mathbf{k} - \mathbf{q}$$

Integration using the properties of the δ-function gives

$$K' = K - \frac{\omega}{c}$$

Solving these two equations gives

$$\begin{aligned} n \cos \theta &= \frac{K}{k} + \frac{\omega}{2kc}(n^2 - 1) \\ &= \beta^{-1} + \frac{\omega\hbar}{2cp}(n^2 - 1) \end{aligned} \tag{13}$$

Further, in going from summation to integration we used the result (Sokolov, 1958)

$$L^{-3} \sum_q \to \frac{1}{(2\pi)^3} \int d^3\mathbf{q}$$

The intensity of radiation then becomes

$$\begin{aligned} W^{\parallel} = \frac{e^2}{c} \int & \frac{\omega\, d\omega}{(1 + 4u_\epsilon r_\epsilon)^{1/2}} \begin{pmatrix} \cos^2\alpha \\ \sin^2\alpha \end{pmatrix} \Bigg\{ \mathcal{L}(n^2 - 1) \\ & + \frac{ss'}{\Lambda}[\mathcal{L}^2(n^2 - 1)(1 - 2\mathcal{L}) + 2\mathcal{L}^2(1 - \beta^2)] \Bigg\} \\ & + \begin{pmatrix} + \\ - \end{pmatrix} \sin 2\alpha \sin \gamma \Bigg\{ s \frac{n\omega\hbar}{cp}\left(1 - \frac{\cos\theta}{\beta n}\right) \\ & + \frac{s'}{\Lambda}\left[\left(1 - \frac{\omega\hbar}{cp\beta}\right)\left(1 + \frac{n\omega\hbar}{cp} - \cos\theta\right) - \frac{\omega\hbar}{cp}\left(\beta^{-1} - \frac{\omega\hbar}{cp}\right)\right]\Bigg\} \\ & + \begin{pmatrix} \sin^2\alpha \\ \cos^2\alpha \end{pmatrix} \Bigg\{ \mathcal{L}^2(n^2 - 1) + \sin^2\theta + \frac{ss'}{\Lambda}\Bigg[\mathcal{L}^2(n^2 - 1)(1 - 2\mathcal{L}) \\ & + 2\mathcal{L}^2(1 - \beta^{-2}) + \left(1 - \frac{\omega\hbar}{cp\beta}\right)\sin^2\theta\Bigg]\Bigg\} \end{aligned} \tag{14}$$

where $\mathcal{L} = \omega\hbar/2cp$ and

$$\Lambda = \left[1 + \left(\frac{\omega\hbar}{cp}\right)^2 - \frac{2\omega\hbar}{cp\beta}\right]^{1/2}$$

From (13) we find for θ

$$\cos^2\theta = \frac{2[(\epsilon_3 - \epsilon_1)/\epsilon_1]\,(\beta^{-1} - \mathcal{L})r_\epsilon - \epsilon_3[1 \mp (1 + 4U_\epsilon r_\epsilon)^{1/2}]}{2[(\epsilon_3 - \epsilon_1)/\epsilon_1]\{\epsilon_3 - [(\epsilon_3 - \epsilon_1)/\epsilon_1](\beta^{-1} - \mathcal{L})^2\}} \qquad (15)$$

The sign is chosen to ensure that

$$0 \leq \cos^2\theta \leq 1$$

Case II. Particle initially moves perpendicular to the optical axis. In this case $k = (k_1, k_2, 0)$. The expression (12) is now a function of θ and φ. It is more convenient to perform the integration first with respect to φ. Again using the properties of the δ-function of complex argument, we obtain

$$\begin{aligned}
W^{\perp} = \frac{e^2}{\pi c}\iint \Delta\omega\, d\omega \sin\theta\, d\theta \\
&\times \begin{pmatrix}\cos^2\alpha \\ \sin^2\alpha\end{pmatrix}\left\{\left[1 + \frac{ss'}{\Lambda}\left(1 - \frac{\omega\hbar}{cp\beta}\right)\right][\mathcal{L}^2(n^2 - 1) + \sin^2\psi]\right. \\
&\left. + 2\frac{ss'}{\Lambda}\mathcal{L}^2(1 - \beta^{-2})\right\} \\
&+ \begin{pmatrix}+ \\ -\end{pmatrix}\sin 2\alpha \cos\gamma\left[s\frac{n\omega\hbar}{cp}\left(1 - \frac{\sin\theta\cos\psi}{\beta n}\right)\right. \\
&+ \frac{s'}{\Lambda}\left(1 - \frac{\omega\hbar}{cp\beta}\right)\left(1 - \sin\theta\cos\psi + \frac{n\omega\hbar}{cp}\right) \\
&\left. - \frac{\omega\hbar}{cp}\left(\beta^{-1} - \frac{\omega\hbar}{cp}\right)\right] \\
&+ \begin{pmatrix}+ \\ -\end{pmatrix}\sin 2\alpha \sin\gamma\left\{\cos\theta \sin 2\psi\left[1 + \frac{s'}{\Lambda}\left(1 - \frac{\omega\hbar}{cp\beta}\right)\right]\right\} \\
&+ \begin{pmatrix}\sin^2\alpha \\ \cos^2\alpha\end{pmatrix}\left\{\left[1 + \frac{ss'}{\Lambda}\left(1 - \frac{\omega\hbar}{cp\beta}\right)\right][\mathcal{L}^2(n^2 - 1) + \cos^2\theta\cos^2\psi]\right. \\
&\left. + 2\frac{ss'}{\Lambda}\mathcal{L}^2(1 - \beta^{-2})\right\} \qquad (16)
\end{aligned}$$

where, instead of (13), we now get

$$n \sin \theta \cos \psi = \beta^{-1} + \frac{\omega\hbar}{2cp} (n^2 - 1) \tag{17}$$

while ψ is connected with φ by

$$k_1 \cos \varphi + k_2 \sin \varphi = k \cos \psi, \qquad k = (k_1^2 + k_2^2)^{1/2}$$

Δ is a function of θ and does not play any role in the construction of the Stokes parameters. Having integrated with respect to φ, the intensity of radiation and so the Stokes parameters become a function of θ.

3. ANALYSIS OF THE POLARIZATION

For the purpose of the analysis we now introduce the Stokes parameters, which we represent in the form of a four-vector:

$$\begin{pmatrix} I \\ \mathbf{P} \end{pmatrix} \tag{18}$$

The components of **P** are defined as follows

$P_1 = +1$ implies plane (linear) polarization along the vector $\boldsymbol{\beta}_2$ (19)

$P_1 = -1$ implies plane (linear) polarization along the vector $\boldsymbol{\beta}_3$ (20)

$P_2 = \pm 1$, plane polarization in a direction making and angle $\pi/4$ (21)
with $\boldsymbol{\beta}_2$ and $\boldsymbol{\beta}_3$, respectively, to the right (22)

$P_3 = +1$, right circular polarization (23)

$P_3 = -1$, left circular polarization

In (18) I is proportional to the intensity of radiation and is used in normalization so that $|\mathbf{P}| \leq 1$.

The constants α and γ in (14) and (16) enable us to identify the elements of the density matrix which are used in the construction of the Stokes parameters. For $\boldsymbol{\beta}_2$ and $\boldsymbol{\beta}_3$, see (5) and (6). We can now pick out the Stokes parameters, but first we average with respect to the final spin states, noting that $\frac{1}{2}\Sigma\, s^1 = 0$.

Case I. Particle moves along the optical axis. In this case the Stokes parameters are given by

$$P_1^{\|} = \frac{-\sin^2\theta}{\sin^2\theta + (\omega\hbar/2cp)^2(n^2 - 1)} \tag{24}$$

$$P_2^{\|} = 0$$

$$P_3^{\|} = \frac{s(n\omega\hbar/cp)[1 - (\cos\theta)/\beta n]}{\sin^2\theta + (\omega\hbar/2cp)^2(n^2 - 1)} \tag{25}$$

Case II. Particle moves perpendicular to the optical axis. Here

$$P_1^{\perp} = \frac{-\sin^2\psi - \cos^2\theta\cos^2\psi}{1 - \sin^2\theta\cos^2\psi + (\omega\hbar/2cp)^2(n^2 - 1)} \tag{26}$$

$$P_2^{\perp} = \frac{2\cos\theta\sin\psi\cos\psi}{1 - \sin^2\theta\cos^2\psi + (\omega\hbar/2cp)^2(n^2 - 1)} \tag{27}$$

$$P_3^{\perp} = \frac{s(n\omega\hbar/cp)[1 - (\sin\theta\cos\psi)/\beta n]}{1 - \sin^2\theta\cos^2\psi + (\omega\hbar/2cp)^2(n^2 - 1)} \tag{28}$$

Expressions (24)–(28) give us the degree of polarization of each component. In addition, if

$$|\mathbf{P}| = (P_1^2 + P_2^2 + P_3^2)^{1/2} = 1$$

we say that the radiation is totally polarized. If, however,

$$|\mathbf{P}| = (P_1^2 + P_2^2 + P_3^2)^{1/2} < 1$$

we say that the radiation is partially polarized. This information is not easy to deduce in their present form. To obtain such information, we consider specific cases, giving definite values to the constants. In case I, θ defines the angle between the direction of motion of the particle and the wave vector. This is not so in case II, where the angle between the direction of motion and the wave vector is defined by $\sin\theta\cos\psi$; taking θ_0 to represent this angle, we have from (17)

$$\sin\theta\cos\psi = \cos\theta_0 = \frac{1}{\beta n} + \frac{n\omega\hbar}{2cp}(1 - n^{-2}) \tag{29}$$

In terms of θ_0, (26)–(28) take the form

$$P_1^{\perp} = \frac{1 - (2\,\mathrm{cosec}^2\theta - 1)\cos^2\theta_0}{\sin^2\theta_0 + (\omega\hbar/2cp)^2(n^2 - 1)} \tag{30}$$

$$P_2^{\perp} = \frac{2\,\mathrm{cosec}^2\theta\cos^2\theta_0(\sin^2\theta - \cos^2\theta_0)^{1/2}}{\sin^2\theta_0 + (\omega\hbar/2cp)^2(n^2 - 1)} \tag{31}$$

$$P_3^+ = \frac{s(n\omega\hbar/cp)[1 - (\cos\theta_0)/\beta n]}{\sin^2\theta_0 + (\omega\hbar/2cp)^2(n^2 - 1)} \tag{32}$$

The forms these expressions now take can be compared with (24) and (25), particularly (25). Further, (25) and (32) should be compared with (1). The role of the spin also needs to be noted.

The values which θ may take are, however, restricted on one hand by the conditions imposed by the δ-function [see (17)] and, as can be seen, from (31), which stipulates

$$\sin\theta \geq \cos\theta_0 \tag{33}$$

In previous work on the energy loss it was shown that radiation occurs under certain conditions which ensure that the energy is a real quantity. One of those conditions is that

$$t^2 = n^2\cos^2\theta \geq b^2$$

where

$$b^2 = \left(\frac{\epsilon_3/2\epsilon_1 + r_\epsilon U_\epsilon}{U_\epsilon^2}\right)\left\{\left[1 + \frac{U_2^2(\epsilon_3 - r_\epsilon^2)}{(\epsilon_3/2\epsilon_1 - r_\epsilon U_\epsilon)^2}\right]^{1/2} - 1\right\} \tag{34}$$

It is interesting to note that these two conditions (33) and (34) reduce to the same inequality, namely

$$\sin^2\theta \geq \sin^2\delta \qquad \text{for some} \quad \theta = \delta$$

where

$$\sin^2\delta = \frac{\epsilon_1\epsilon_3^2 + 2\epsilon_3(\epsilon_3 - \epsilon_1)(1/\beta - \omega\hbar/2cp)r_0 \pm \epsilon_1^2\epsilon_3(1 + 4U_\epsilon r_\epsilon)^{1/2}}{2[(\epsilon_3 - \epsilon_1)^2(1/\beta - \omega\hbar/2cp)^2 + \epsilon_1\epsilon_3(\epsilon_3 - \epsilon_1)]} \tag{35}$$

$$r_\epsilon = \frac{1}{\beta} + \frac{\omega\hbar}{2cp}(\epsilon_3 - 1), \qquad U_\epsilon = \frac{\epsilon_3 - \epsilon_1}{\epsilon_1}\frac{\omega\hbar}{2cp}$$

$$r_0 = \frac{1}{\beta} + \frac{\omega\hbar}{2cp}(\epsilon_1 - 1)$$

This expression asserts that the two cones (when the particle moves parallel and perpendicular, respectively, to the optical axis) have no points of intersection. Thus in case II,

$$\delta \leq \theta \leq \pi/2$$

and δ thus determines the threshold of radiation. This implies that unlike case I, where θ takes a definite value, here θ is smeared out over a small interval. The polarization will therefore vary with θ in this small interval and the whole radiation so to speak forms a halo about the axis of the cone (the axis along which the particle moves).

4. DISCUSSION AND CONCLUSION

The essential difference between the two cases is that when the particle moves along the optical axis, the quanta are emitted at a fixed angle to the optical axis, forming a hollow cone, whereas when the particle moves perpendicular to the optical axis, the radiation is smeared out within a small band forming a halo about the axis of the cone. The former resembles the case of an isotropic medium. In this case the polarization vector is constant. In the latter case the direction of the vector is fixed, but the magnitude varies with the angle in the interval in which the radiation occurs.

A clearer picture is obtained if we take some definite values. We consider a medium in which $\epsilon_3 = 3$, $\epsilon_1 = 2$, $\beta = 0.8$, and $\omega\hbar/cp = 0.1$. This gives for the first case

$$\cos\theta = 0.824$$

so that

$$P_1^{\parallel} = 0.954$$
$$P_2^{\parallel} = 0$$
$$P_3^{\parallel} = 0.14$$
$$(P_1^{\parallel})^2 + (P_2^{\parallel})^2 + (P_3^{\parallel})^2 = 1$$

The sign in (24) means the vector of linear polarization is along $\boldsymbol{\beta}_3$. Thus the radiation is totally polarized.

The sign of the circular polarization is completely determined by the spin ($S = \pm 1$).

In the second case, when the particle moves perpendicular to the optical axis,

$$0.8 \leq \sin^2\theta \leq 1$$

The angle of the cone is about 7° and $36° \leq \theta_0 \leq 43°18'$. The radiation is confined within this interval. The change of polarization with the angle is also limited to this region. The change of the components of the degree of polarization is exhibited in Fig. 1. $P_3^{\perp}$ is of small magnitude and varies slowly. $P_1^{\perp}$ is along the vector $\boldsymbol{\beta}_2$ and $P_2^{\perp}$ to the right of this. Making allowance for the approximations used in the computations, we find again that generally

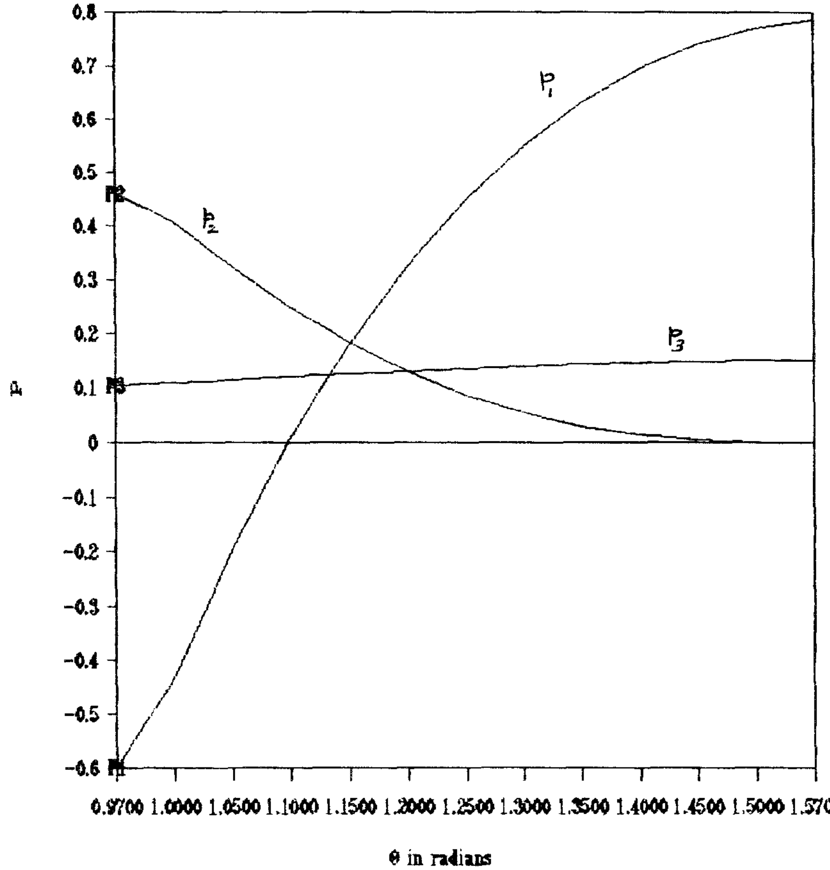

Fig. 1. Degree of polarization. P_1, linear polarization; P_2 linear polarization $\pi/4$ to P_1; P_3 circular polarization.

$$[(P_1^{\perp})^2 + (P_2^{\perp})^2 + (P_3^{\perp})^2]^{1/2} = 1$$

So here also radiation is totally polarized. As has been remarked, in both cases, circular polarization is purely a quantum effect and its sign depends on the polarization of the particle. McMaster (1954) remarked that in optics a Nicol prism is not sensitive to circular polarization and that circular polarization must first be transformed into plane polarization by means of a quarter-wave plate before measurement can be made. We may therefore infer from this study that theoretically, the method of quantum electrodynamics provides the quarter-wave plate which enables us to detect circular polarization. The formulas (25) and (32) definitely show that when $\hbar \to 0$, these components also vanish. Their comparatively small value, as seen from the calculated values, is also to be noted.

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